IV. *A Proposition relating to the Combination of* Transparent Lens's *with* Reflecting Planes. *By* J. Hadley, *Esq; V. Pr. R. S. Communicated to the* Royal Society, January 9, 1734.

HAVING proposed the using of a Telescope with the Instrument for taking Angles, which I formerly laid before this Society, (*See* N° 420.) it gave me Occasion to consider the Effects of the combining several Kinds of Telescopes with reflecting Planes, and, among others, led me to the following Proposition.

That if two Lens's of equal focal Length be put together in the Form of a Telescope, and a Plane Speculum be placed before one of them, so that the Axis of the Telescope make any Angle with its Surface, and a Ray of Light (the Line of whose Direction lies in a Plane perpendicular to that Surface, and passing through the Axis of the Telescope) fall on it, and be reflected from it, so as to pass through the Telescope; then the Line of its last Direction, after passing the Telescope, will make an Angle with that of its first Direction, before its Incidence on the Speculum, very nearly equal to double the Angle made between the Axis of the Telescope, and the Surface of the Speculum.

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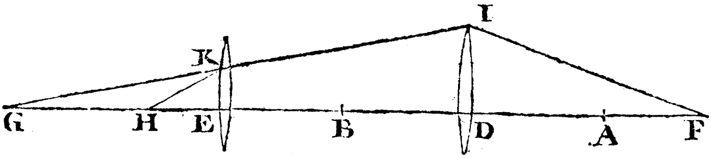
Let the Line FG be the common Axis of the two Lens's ID and KE, of equal focal Lengths; to which let the Lines AD, DB and BE, be each equal; and let a Ray of Light, issuing from a Point in the Axis F, fall on the Lens ID at I, and be there refracted into the Line IG, cutting the Axis in G, and meeting the Lens KE in K, where let the Ray be again refracted into the Line KH, cutting the aforesaid Axis in H: The Angles IFD and KHE are very nearly equal.

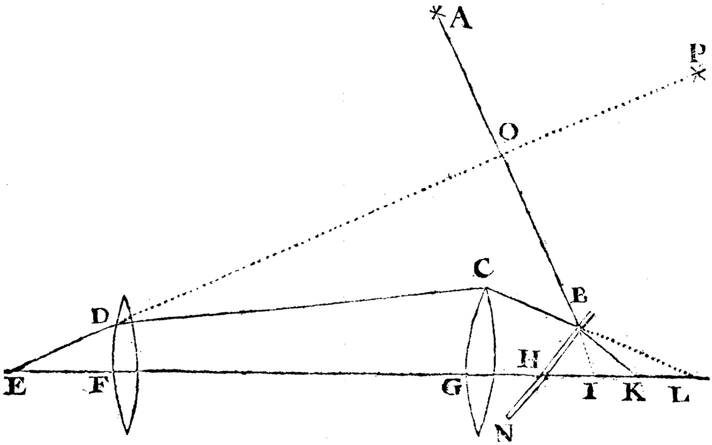


DEMONSTRATION.

It is known from Doptricks, that the Lines FI, IG, KH, and FG, are all in the same Plane; and by the Construction the Lines AD, DB, and BE are equal; and by *Propos.* 20 of Huygens's *Dioptricks*, the Lines FA, FD, and FG are continually proportional; and consequently FA is to AD as FD to DG, and dividing, FA is to AD as FD — FA (= to AD) is to DG — AD (= to BG.) Therefore AD is to BG as FD to DG. By the same *Proposition*, the Lines BG, EG, and HG are also continually proportional, and BE (= to AD) is to BG as EH is to EG. Hence it follows, that the Lines FD, DG, and EH, EG, are

are Proportionals. But as F D is to D G, ſo is the Tangent of the Angle I G D or K G E to the Tangent of the Angle I F D; and as E H is to E G, ſo is the Tangent of the Angle K G E to the Tangent of the Angle K H E. The Tangent of the Angle K G E therefore has the ſame Proportion to the Tangents of each of the Angles I F D and K H E, and conſequently thoſe Angles are equal. *Q. E. D.*

N. B. In the Demonſtration of the above-cited Propoſition of *Huygens*, the Thickneſs of the Lens's are neglected, and the Diſtance of the Points I and K, from the Line F G, ſuppoſed very ſmall; ſo that if either of thoſe are too great, there may ariſe a ſenſible Difference between the Angles I F D and K H E.



Let D F and C G repreſent the two Lens's put together as before, having their common Axis in the Line E L, and B N a plane Speculum to which that Line is inclined in the Angle G H N, and let

A B

A B be a Ray of Light falling on the Speculum at B, as is before expressed, and let it be there reflected towards the Point C of the Lens C G, where it is refracted towards the Point D of the Lens D F, and there again refracted into the Line D E, cutting the Axis in E. The Angle A O P contained between this last Line D E, continued backwards, and the first Line of Incidence of the Ray A B, will be very nearly equal to double the Angle of Inclination of the Axis of the Lens's E L to the Plane of the Speculum B N; *i. e.* double the Angle G H N.

DEMONSTRATION.

Produce the Lines of Incidence and Reflection of the Ray A B and B C, 'till they meet the Axis of the two Lens's in I and L; and through the Point B draw B K perpendicular to the Plane of the Speculum, and cutting the same Axis in K, the Angles K B L and K B I are equal. The Angle K L B is the Difference of the Angles I K B and K B L; and the Angle H I B is the Sum of the Angles I K B and K B I (equal to K B L): Therefore the Angle I K B is equal to half the Sum of the Angles H I B and K L B. But by the 'foregoing *Lemma*, the Angles K L B and F E D are very nearly equal. Therefore the Angle I K B is nearly equal to half the Sum of the Angles H I B and F E D; that is, to half the Angle P O B, and its Complement B H I or G H N is nearly equal to half the Angle A O P the Complement of P O B to a Semicircle. *Q. E. D.*

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If the first Incidence of the Ray be ſuppoſed to be in the Line E D, it will proceed in the ſame Track as before, but with the contrary Directions; ſo that the Angle E O B made between the firſt incident Ray and the laſt reflected, will ſtill be equal to the Double of G H N, as before.

It is evident that on this Principle an Inſtrument might be conſtructed, the Effects of which would in a great Meaſure reſemble thoſe of that before mentioned (N° 420): But it would be liable to the Errors ariſing both from the ſpherical Figure of the Lens's, and alſo the different Refrangibility of the Rays of Light, when the Object is ſeen at a Diſtance from the Axis of the Teleſcope; altho' thoſe Errors, by a proper Diſpoſition of the Parts of the Inſtrument, may be reduced to a very ſmall Quantity. However, for this Reaſon, and alſo becauſe the Inſtrument ſeemed to me to be attended with greater Inconveniencies, both in its Conſtruction and Uſe, than the other, I have not thought it neceſſary to give any more particular Deſcription of it.

V. *An Account of a large* Bony Subſtance *found in the* Womb, *which was ſhewn to the* Royal Society, May *the* 17*th*, 1733, *by* Edward Hody, *M. D. F. R. S.*

E——— S———, aged 57, died in St. *James's Market*, *January*, 1725. In examining the *Pelvis* of this Woman, I found a large *Bony-Sub-*